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Form factors of $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ transitions and light-cone sum rules

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Abstract. The method of light-cone QCD sum rules is applied to the calculation of the form factors of $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ transitions. We consider the dispersion relation for the $\gamma^*(Q^2)\gamma^*(q^2) \to \pi^0$ amplitude in the variable q^2. At large virtualities q^2 and Q^2, this amplitude is calculated in terms of light-cone wave functions of the pion. As a next step, the light-cone sum rule for the $\gamma^*(Q^2)\rho \to \pi$ form factor is derived. This sum rule, together with the quark-hadron duality, provides an estimate of the hadronic spectral density in the dispersion relation. Finally, the $\gamma^*(Q^2)\gamma \to \pi^0$ form factor is obtained taking the $q^2 = 0$ limit in this relation. Our predictions are valid at $Q^2 \geq 1$ GeV^2 and have a correct asymptotic behaviour at large Q^2.

1 Introduction

Light-cone wave functions (distribution amplitudes) of hadrons have been introduced in QCD to define the long-distance part of exclusive processes with large momentum transfer [1,2]. The same wave functions serve as an input in QCD light-cone sum rules [3–8] which are based on the light-cone operator product expansion (OPE) of vacuum-hadron correlators. At asymptotically large normalization scale, the light-cone wave functions are given by perturbative QCD. To estimate or at least to constrain nonasymptotic corrections, one needs either nonperturbative methods or, in a more direct way, measurements of hadronic quantities which are sensitive to the shape of light-cone wave functions.

One of the simplest processes determined by the light-cone wave functions of the pion is the transition $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$ of two virtual photons into a neutral pion. This process is defined by the matrix element

$$\int d^4x e^{-iq_1x} \langle \pi^0(p) \mid T\{j_\mu(x)j_\nu(0)\} \mid 0\rangle = i\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta F^{\gamma^*\pi}(Q^2,q^2) \ , \tag{1}$$

where $Q^2 = -q_1^2$, $q^2 = -q_2^2$ are the virtualities of the photons and $j_\mu = (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d)$ is the quark electromagnetic current. If both Q^2 and q^2 are sufficiently large, the T-product of currents in (1) can be expanded near the light-cone $x^2 = 0$. The leading term of this expansion yields [1]:

$$F^{\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}f_\pi}{3} \int_0^1 \frac{du\ \varphi_\pi(u)}{Q^2(1-u)+q^2u} \ , \tag{2}$$

where $\varphi_\pi(u)$ is the pion wave function of twist 2. Nonleading terms of the light-cone OPE are determined by pion wave functions of higher twist. Their contributions to $F^{\gamma^*\pi}$ are suppressed by additional inverse powers of photon virtualities. Therefore, measurements of the form factor $F^{\gamma^*\pi}(Q^2,q^2)$ at large Q^2 and $q^2 \neq Q^2$ will be a direct source of information on $\varphi_\pi(u)$.

Recently, the CLEO collaboration has measured [9] the photon-pion transition form factor $F^{\gamma\pi}(Q^2) \equiv F^{\gamma^*\pi}(Q^2,0)$, where one of the photons is nearly on-shell and the other one is highly off-shell, with the virtuality in the range 1 GeV^2 $< Q^2 <$ 10 GeV^2. A straightforward calculation of $F^{\gamma\pi}(Q^2)$ in QCD is, however, not possible. In particular, at $q^2 \to 0$, it is not sufficient to retain a few terms of the light-cone OPE of (1). One has, in addition, to take into account the interaction of the small-virtuality photon at long distances of $O(1/\sqrt{q^2})$ (for a recent discussion, see [10,11]).

In this paper, a simple method is suggested to calculate the form factor $F^{\gamma\pi}(Q^2)$ at sufficiently large Q^2 (practically, at $Q^2 \geq 1$ GeV^2), in terms of the pion light-cone wave functions. The method allows to avoid the problem of the photon long-distance interaction by performing all QCD calculations at sufficiently large q^2. In parallel, the form factor of the $\gamma^*\rho \to \pi$ transition is obtained from the light-cone sum rule. In the following sections, the calculational procedure is described, the light-cone OPE of the amplitude (1) is performed up to twist 4 and the numerical results for the $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ transition form

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factors are presented and discussed. The account of $O(\alpha_s)$ corrections is postponed to a future study.

2 The method

Our starting object is the dispersion relation for the amplitude $F^{\gamma^*\pi}(Q^2, q^2)$ in the variable q^2 and at fixed large Q^2. Physical states in the q^2–channel include vector mesons $\rho, \omega, \rho', \omega', ...$ and a continuum of hadronic states with the same quantum numbers. We assume that the spectral density in the dispersion relation can be approximated by the ground states ρ, ω and the higher states with an effective threshold s_0:

$$F^{\gamma^*\pi}(Q^2, q^2) = \frac{\sqrt{2} f_\rho F^{\rho\pi}(Q^2)}{m_\rho^2 + q^2} + \int\limits_{s_0}^{\infty} ds\ \frac{\rho^h(Q^2, s)}{s + q^2}. \quad (3)$$

Here, the ρ and ω contributions are combined in one resonance term assuming $m_\rho \simeq m_\omega$, adopting zero-width approximation and defining the matrix elements of electromagnetic transitions

$$\frac{1}{3}\langle \pi^0(p) \mid j_\mu \mid \omega(q_2)\rangle \simeq \langle \pi^0(p) \mid j_\mu \mid \rho^0(q_2)\rangle = F^{\rho\pi}(Q^2) m_\rho^{-1} \epsilon_{\mu\nu\alpha\beta} e^\nu q_1^\alpha q_2^\beta\ , \quad (4)$$

and the decay constants of vector mesons

$$3\langle \omega \mid j_\nu \mid 0\rangle \simeq \langle \rho^0 \mid j_\nu \mid 0\rangle = \frac{f_\rho}{\sqrt{2}} m_\rho e_\nu^*\ , \quad (5)$$

e_ν being the polarization vector of the ρ-meson. Approximate relations in (4) and (5) follow from the quark content of ω and ρ and from the isospin symmetry.

Two important points should be emphasized. First, the dispersion relation (3) does not contain subtraction terms [12]. Otherwise, at $q^2 \to \infty$, the asymptotic behaviour of $F^{\gamma^*\pi}(Q^2, q^2)$ dictated by (2) will be violated. Second, due to absence of massless resonances, it is possible to analytically continue (3) to $q^2 = 0$. One then obtains the form factor $F^{\gamma\pi}(Q^2)$. Therefore, the outlined problem can be solved if the function $F^{\rho\pi}(Q^2)$ and the integral over $\rho^h(Q^2, s)$ in the dispersion relation (3) are known[1].

To estimate the spectral density $\rho^h(Q^2, s)$ of the higher states in (3), we employ the usual quark-hadron duality:

$$\rho^h(Q^2, s) = \frac{1}{\pi} \mathrm{Im} F_{QCD}^{\gamma^*\pi}(Q^2, s) \Theta(s - s_0)\ , \quad (6)$$

where $F_{QCD}^{\gamma^*\pi}$ is the amplitude (1) calculated in QCD using the light-cone OPE. The form factor of the $\gamma^*\rho \to \pi$ transition determining the residue of the resonance term in (3) can also be obtained in the same framework. We follow the procedure described in [6,7] and [8] where the $B \to \pi$ form factor and the pion electromagnetic form factor have been calculated, respectively. One equates the dispersion relation (3) with $F_{QCD}^{\gamma^*\pi}(Q^2, q^2)$ at large q^2, where the light-cone OPE is reliable and higher-twist terms are under quantitative control:

$$\frac{\sqrt{2} f_\rho F^{\rho\pi}(Q^2)}{m_\rho^2 + q^2} + \int\limits_{s_0}^{\infty} ds\ \frac{\rho^h(Q^2, s)}{s + q^2} = \frac{1}{\pi} \int\limits_0^{\infty} ds\ \frac{\mathrm{Im} F_{QCD}^{\gamma^*\pi}(Q^2, s)}{s + q^2}\ . \quad (7)$$

Using (6), subtracting the integral over $\rho^h(Q^2, s)$ from the dispersion integral on the r.h.s. of (7) and performing the Borel transformation in q^2 yields the light-cone sum rule

$$\sqrt{2} f_\rho\ F^{\rho\pi}(Q^2) = \frac{1}{\pi} \int\limits_0^{s_0} ds\ \mathrm{Im} F_{QCD}^{\gamma^*\pi}(Q^2, s) \times \exp\left(\frac{m_\rho^2 - s}{M^2}\right)\ . \quad (8)$$

Substituting (8) and the duality approximation (6) in the initial dispersion relation (3), and, finally, taking the $q^2 \to 0$ limit we obtain an estimate of the $\gamma^*\gamma \to \pi^0$ form factor:

$$F^{\gamma\pi}(Q^2) = \frac{1}{\pi m_\rho^2} \int\limits_0^{s_0} ds\ \mathrm{Im} F_{QCD}^{\gamma^*\pi}(Q^2, s) \exp\left(\frac{m_\rho^2 - s}{M^2}\right) + \frac{1}{\pi} \int\limits_{s_0}^{\infty} \frac{ds}{s} \mathrm{Im} F_{QCD}^{\gamma^*\pi}(Q^2, s)\ . \quad (9)$$

3 Light-cone expansion

It remains to calculate the amplitude $F^{\gamma^*\pi}(Q^2, q^2)$ using light-cone OPE and to take its imaginary part. The procedure essentially follows [4,6–8] where vacuum-pion correlators similar to the amplitude (1) have been calculated[2].

To obtain the contribution of two-particle (quark-antiquark) wave functions of the pion, one has to contract two quark fields in the product of currents in (1):

$$\int d^4x e^{-iq_1 x} \langle \pi^0(p) \mid T\{j_\mu(x) j_\nu(0)\} \mid 0\rangle = 2 \int d^4x e^{-iq_1 x} \times \langle \pi^0(p) \mid \left(\frac{2}{3}\right)^2 \bar{u}(x) \gamma_\mu i\hat{S}_u(x, 0) \gamma_\nu u(0) + \left(\frac{1}{3}\right)^2 \bar{d}(x) \gamma_\mu i\hat{S}_d(x, 0) \gamma_\nu d(0) \mid 0\rangle\ , \quad (10)$$

[1] A similar approach was used in [13] to estimate the structure function of the real photon. The dispersion relation for the structure function of the virtual photon was analytically continued to the zero virtuality limit

[2] The light-cone OPE of the amplitude (1) was also studied in [14] using different definitions of the higher-twist wave functions

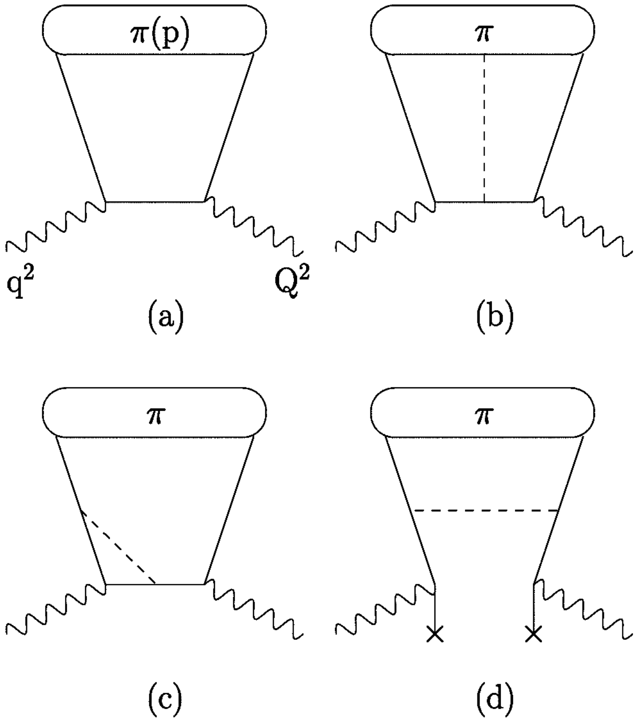

Fig. 1a–d. Diagrams corresponding to the light-cone OPE of the amplitude (1). Solid lines represent quarks, dashed lines gluons, wavy lines electromagnetic currents. The ovals denote light-cone wave functions of the pion

and substitute the free-quark propagator

$$i\hat{S}_q^0(x,0) = \langle 0 \mid T\{q(x)\bar{q}(0)\} \mid 0\rangle = \frac{i\,\not{x}}{2\pi^2 x^4}. \quad (11)$$

The light quark masses and the pion mass are neglected in this calculation ($p^2 = m_\pi^2 \simeq 0$). The approximation (10) corresponds to the diagram shown in Fig. 1a. The factor 2 takes into account two equal contributions of this diagram with opposite directions of quark lines. The matrix elements of nonlocal quark-antiquark operators emerging in (10) are expanded near the light-cone:

$$\begin{aligned}
&\langle\pi^0(p)|\bar{u}(x)\gamma_\mu\gamma_5 u(0)|0\rangle \\
&= -\langle\pi^0(p)|\bar{d}(x)\gamma_\mu\gamma_5 d(0)|0\rangle \\
&= -ip_\mu\frac{f_\pi}{\sqrt{2}}\int_0^1 du\, e^{iup\cdot x}\left(\varphi_\pi(u) + x^2 g_1(u)\right) \\
&\quad + \frac{f_\pi}{\sqrt{2}}\left(x_\mu - \frac{x^2 p_\mu}{p\cdot x}\right)\int_0^1 du\, e^{iup\cdot x} g_2(u)\ , \quad (12)
\end{aligned}$$

where $\varphi_\pi(u)$ and $g_{1,2}(u)$ are the twist 2 and twist 4 wave functions of the pion, respectively. The twist 3 terms of the light-cone OPE of the amplitude (1) are proportional to m_π^2 and therefore vanish in the adopted chiral limit. Terms corresponding to twists higher than 4 are neglected. The light-cone gauge is assumed for the gluon field suppressing the path-ordered gauge factors in the matrix elements (12). To twist 4 accuracy, the result for the diagram Fig. 1a reads:

$$F_{(a)}^{\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}f_\pi}{3}\left(\int_0^1 \frac{du\ \varphi_\pi(u)}{Q^2(1-u)+q^2u} - 4\int_0^1 \frac{du\ (g_1(u)+G_2(u))}{(Q^2(1-u)+q^2u)^2}\right), \quad (13)$$

where $G_2(u) = -\int_0^u dv\ g_2(v)$. The first, leading term was already given in (2).

Furthermore, there are contributions to the light-cone OPE due to many-particle (higher Fock) states in the pion. With the same accuracy, one has to include the quark-antiquark-gluon wave functions taking into account the gluon emission from the virtual quark (Fig. 1b). In order to obtain this contribution, the quark propagator including the interaction with gluons in first order:

$$i\hat{S}_q^G(x,0) = -\frac{ig_s}{16\pi^2x^2} \times \int_0^1 dv\,(\not{x}\sigma_{\alpha\beta} - 4ivx_\alpha\gamma_\beta)\,G^{\alpha\beta}(vx) \quad (14)$$

with $G_{\alpha\beta} = G^a_{\alpha\beta}\frac{\lambda^a}{2}$, should be substituted in (10). One then encounters matrix elements of nonlocal quark-antiquark-gluon operators. They are defined in [4,15]:

$$\begin{aligned}
&\langle\pi^0(p)|\bar{u}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)u(0)|0\rangle \\
&= -\langle\pi^0(p)|\bar{d}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)d(0)|0\rangle \\
&= \frac{f_\pi}{\sqrt{2}}\left\{\left[p_\beta\left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{p\cdot x}\right) - p_\alpha\left(g_{\beta\mu} - \frac{x_\beta p_\mu}{p\cdot x}\right)\right]\right. \\
&\quad \times\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i)e^{ip\cdot x(\alpha_1+v\alpha_3)} + \frac{p_\mu}{p\cdot x}(p_\alpha x_\beta - p_\beta x_\alpha) \\
&\quad \left.\times\int \mathcal{D}\alpha_i\ \varphi_\parallel(\alpha_i)e^{ip\cdot x(\alpha_1+v\alpha_3)}\right\}, \quad (15)
\end{aligned}$$

$$\begin{aligned}
&\langle\pi^0(p)|\bar{u}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)u(0)|0\rangle \\
&= -\langle\pi^0(p)|\bar{d}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)d(0)|0\rangle \\
&= \frac{if_\pi}{\sqrt{2}}\left\{\left[p_\beta\left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{p\cdot x}\right) - p_\alpha\left(g_{\beta\mu} - \frac{x_\beta p_\mu}{p\cdot x}\right)\right]\right. \\
&\quad \times\int \mathcal{D}\alpha_i\ \tilde{\varphi}_\perp(\alpha_i)e^{ip\cdot x(\alpha_1+v\alpha_3)} + \frac{p_\mu}{p\cdot x}(p_\alpha x_\beta - p_\beta x_\alpha) \\
&\quad \left.\times\int \mathcal{D}\alpha_i\ \tilde{\varphi}_\parallel(\alpha_i)e^{ip\cdot x(\alpha_1+v\alpha_3)}\right\}, \quad (16)
\end{aligned}$$

where $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\sigma\tau}G^{\sigma\tau}$ and $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$. The wave functions $\varphi_\perp$, $\varphi_\parallel$, $\tilde{\varphi}_\perp$ and $\tilde{\varphi}_\parallel$ have twist 4. Using (15) and (16) and integrating (10) over x, one obtains the answer for the diagram Fig. 1b:

$$
\begin{aligned}
&F_{(b)}^{\gamma^*\pi}(Q^2,q^2) \\
&= -\frac{\sqrt{2}f_\pi}{3}\int\limits_0^1 \frac{du}{(Q^2(1-u)+q^2u)^2}\int\limits_0^u d\alpha_1 \int\limits_0^{1-u}\frac{d\alpha_2}{\alpha_3} \\
&\times\Bigg(\frac{1-2u+\alpha_1-\alpha_2}{\alpha_3}\varphi_\parallel(\alpha_1,\alpha_2,\alpha_3) \\
&-\widetilde{\varphi}_\parallel(\alpha_1,\alpha_2,\alpha_3)\Bigg)_{\alpha_3=1-\alpha_1-\alpha_2} . \qquad (17)
\end{aligned}
$$

The wave functions $\varphi_\perp$ and $\tilde{\varphi}_\perp$ drop out due to the antisymmetry of the amplitude (1) in μ,ν.

Taking the sum of (13) and (17) and transforming the integration variable, $u \to Q^2/(s+Q^2)$, one obtains the OPE result for the amplitude (1) in the form of a dispersion integral:

$$
\begin{aligned}
F_{QCD}^{\gamma^*\pi}(Q^2,q^2) &= \frac{\sqrt{2}f_\pi}{3}\int\limits_0^1 \frac{du}{Q^2(1-u)+q^2u} \\
&\times\left(\varphi_\pi(u)-\frac{\varphi^{(4)}(u)}{Q^2(1-u)+q^2u}\right) \\
&= \frac{1}{\pi}\int\limits_0^\infty ds\,\frac{\mathrm{Im}F_{QCD}^{\gamma^*\pi}(Q^2,s)}{s+q^2} \qquad (18)
\end{aligned}
$$

with the imaginary part

$$
\begin{aligned}
&\frac{1}{\pi}\mathrm{Im}F_{QCD}^{\gamma^*\pi}(Q^2,s) \\
&= \frac{\sqrt{2}f_\pi}{3}\left(\frac{\varphi_\pi(u)}{s+Q^2}-\frac{1}{Q^2}\frac{d\varphi^{(4)}(u)}{ds}\right)_{u=\frac{Q^2}{s+Q^2}} , \qquad (19)
\end{aligned}
$$

where the following combination of twist 4 wave functions is introduced:

$$
\begin{aligned}
\varphi^{(4)}(u) &= 4\left(g_1(u)+G_2(u)\right)+\int\limits_0^u d\alpha_1 \int\limits_0^{1-u}\frac{d\alpha_2}{\alpha_3} \\
&\times\Bigg(\frac{1-2u+\alpha_1-\alpha_2}{\alpha_3}\varphi_\parallel(\alpha_1,\alpha_2,\alpha_3) \\
&-\widetilde{\varphi}_\parallel(\alpha_1,\alpha_2,\alpha_3)\Bigg)_{\alpha_3=1-\alpha_1-\alpha_2} . \qquad (20)
\end{aligned}
$$

The twist 2 wave function can be expanded [1,2] in Gegenbauer polynomials $C_n^{3/2}$:

$$
\varphi_\pi(u,\mu) = 6u(1-u)\Big[1+\sum_{n=2,4,\dots} a_n(\mu)C_n^{3/2}(2u-1)\Big], \qquad (21)
$$

Nonperturbative effects are contained in the coefficients a_n which logarithmically depend on the normalization scale μ of the wave function. Substituting in (20) the asymptotic twist 4 wave functions from [15] we obtain a simple expression:

$$
\varphi^{(4)}(u,\mu) = \frac{80}{3}\delta^2(\mu)u^2(1-u)^2 , \qquad (22)
$$

where the parameter δ^2 determines the matrix element

$$
\langle \pi(p)|g_s\bar{d}\tilde{G}_{\alpha\mu}\gamma^\alpha u|0\rangle = i\delta^2 f_\pi p_\mu . \qquad (23)
$$

The nonasymptotic corrections to (22) are not shown for brevity. At $Q^2 = q^2$, the integrals over wave functions in (18) convert into normalization factors and the light-cone OPE is reduced to the short-distance expansion. The amplitude $F_{QCD}^{\gamma^*\pi}$ then simplifies:

$$
F_{QCD}^{\gamma^*\pi}(Q^2,Q^2) = \frac{\sqrt{2}f_\pi}{3Q^2}\left(1-\frac{8}{9}\frac{\delta^2}{Q^2}\right), \qquad (24)
$$

coinciding with the result of the short-distance expansion obtained in [16].

The $O(\alpha_s)$ corrections to $F_{QCD}^{\gamma^*\pi}$ are beyound the scope of the present paper. Nevertheless, a few comments are in order. The perturbative α_s–correction to the leading twist 2 term (2) has been calculated in [17]. One of the relevant diagrams is shown in Fig. 1c. In our approach, the account of this effect requires a calculation of the imaginary part of the $O(\alpha_s)$-amplitude obtained in [17]. Simultaneously, the scale-dependence of the wave function (21) should be taken into account in the next-to-leading order. The perturbative correction to the twist-4 contribution is unknown but is most likely inessential. Furthermore, one has to take into account the $O(\alpha_s)$ contributions of four-quark operators to $F_{QCD}^{\gamma^*\pi}$. They were studied in [4] and in [14]. However, the results differ, calling for a new, independent calculation. The nonlocal four-quark matrix elements have been approximated by factorizing two quark operators and taking their vacuum average $\langle\bar{q}q\rangle$. The remaining two operators then form a pion wave function of twist 3. One of the relevant diagrams is shown in Fig. 1d. Schematically, the corresponding correction to $F_{QCD}^{\gamma^*\pi}$ is

$$
F_{(d)}^{\gamma^*\pi}(Q^2,q^2) \sim \frac{\alpha_s\langle\bar{q}q\rangle}{Q^2q^2}\int\limits_0^1 \frac{du\;\varphi_{tw3}(u)}{Q^2(1-u)+q^2u} . \qquad (25)
$$

The divergence of (25) at $q^2 \to 0$ clearly signals that a truncated light-cone OPE is not applicable at small q^2, even if Q^2 is large. In the full answer, this and similar divergences should cancel with additional nonperturbative contributions corresponding to long-distance interactions of the photon. The latter can be taken into account by introducing the photon light-cone wave function. For the short-distance OPE, such cancellation was studied in [10]. The approach used here avoids this problem, because the hadronic dispersion relation is approximated by the light-cone OPE at sufficiently large q^2, where the terms similar to (25) are suppressed.

It remains now to substitute in (8) and (9) the obtained expression (19) for the imaginary part $\mathrm{Im}F_{QCD}^{\gamma^*\pi}$.

Returning to the integration variable u one finally obtains the $\gamma^*\rho \to \pi^0$ form factor

$$F^{\rho\pi}(Q^2) = \frac{f_\pi}{3f_\rho} V(Q^2, M^2) \;, \qquad (26)$$

and the $\gamma^*\gamma \to \pi^0$ form factor

$$Q^2 F^{\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \left(\frac{Q^2}{m_\rho^2} V(Q^2, M^2) + H(Q^2) \right) \;, \qquad (27)$$

where

$$V(Q^2, M^2) = \int_{\frac{Q^2}{s_0+Q^2}}^{1} \frac{du}{u} \left(\varphi_\pi(u) + \frac{u}{Q^2} \frac{d\varphi^{(4)}(u)}{du} \right) \times \exp\left(-\frac{Q^2(1-u)}{uM^2} + \frac{m_\rho^2}{M^2} \right) \qquad (28)$$

and

$$H(Q^2) = \int_0^{\frac{Q^2}{s_0+Q^2}} \frac{du}{1-u} \left(\varphi_\pi(u) + \frac{u}{Q^2} \frac{d\varphi^{(4)}(u)}{du} \right) . \qquad (29)$$

One should emphasize that the light-cone sum rule (26) takes into account soft (end-point) contributions to the $\gamma^*\rho \to \pi$ form factor yielding $F^{\rho\pi}(Q^2) \sim 1/Q^4$ at $Q^2 \to \infty$ (for a more general discussion see [8]). In order to account for the hard-gluon exchange mechanism, which becomes important at large momentum transfer, one should include the perturbative α_s-correction in the sum rule.

In the dispersion relation (27), the resonance part proportional to $V(Q^2)$ vanishes at $Q^2 \to \infty$ and $F^{\gamma\pi}(Q^2) \sim 1/Q^2$, in accordance with (2). At moderate $Q^2 \sim 1$ GeV^2, the contributions from the vector meson and higher states are of the same order.

4 Numerical results

In order to proceed to the numerical analysis of the sum rule (26) and relation (27), one has to specify the input. We take $f_\pi = 132$ MeV, $m_\rho = 770$ MeV and $f_\rho = 216$ MeV. The latter value is obtained from (5) and the $\rho^0 \to e^+e^-$ width [18]. The threshold parameter $s_0 = 1.5$ GeV^2 is determined from the two-point sum rule in the ρ-meson channel [19]. The value of $\delta^2(1\text{ GeV}) = 0.2$ GeV^2 has been estimated from the corresponding sum rules in [16,20]. Furthermore, we consider three different approximations for the twist 2 wave function (21): the asymptotic wave function ($a_n = 0$), the CZ-wave function [21] ($a_2(\mu_0) = 2/3$, $a_{n>2} = 0$) and the BF-wave function [4] ($a_2(\mu_0) = 2/3$, $a_4(\mu_0) = 0.43$, $a_{n>4} = 0$), where $\mu_0 = 0.5$ GeV. The nonasymptotic corrections to the twist 4 wave functions entering (20) have been roughly estimated in [15]. Including them, one obtains negligible changes of the numerical results. Hence, uncertainties of these corrections play no role here. Finally, the leading-order evolution of $\varphi_\pi(u, \mu)$ and $\delta^2(\mu)$ is taken into account assuming $\mu = \sqrt{Q^2}$.

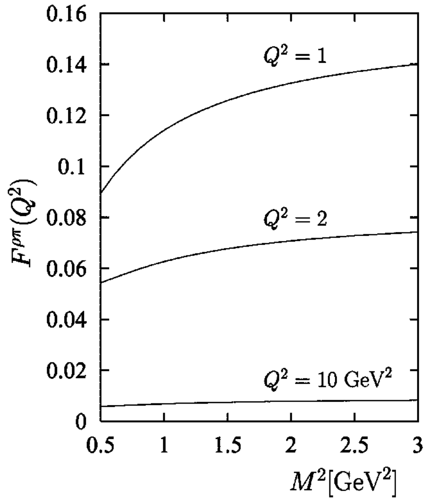

Fig. 2. Form factor of the $\gamma^*\rho \to \pi$ transition obtained from the light-cone sum rule as a function of the Borel parameter at different values of the momentum transfer

In Fig. 2, the form factor $F^{\rho\pi}(Q^2)$ calculated from (26) with the asymptotic $\varphi_\pi(u)$, is plotted as a function of the Borel mass parameter M. In light-cone sum rules, the correlation function is expanded in inverse powers of uM^2, where u is the light-cone momentum fraction, that is the integration variable in (28). To obtain suitable intervals of M in (26), we adopt $M^2 = M_{2pt}^2/\langle u \rangle$, where M_{2pt} is the Borel parameter of the two-point sum rule in the ρ-channel, and calculate the average value $\langle u \rangle$ at each Q^2 separately. We then take $0.5 < M_{2pt}^2 < 0.8$ GeV^2, according to [19]. The resulting interval of M^2 is shifting from $0.9 - 1.6$ GeV^2 at $Q^2 \sim 1$ GeV^2 to $0.5 - 0.9$ GeV^2 at $Q^2 = 10$ GeV^2. Within all these intervals, the twist 4 part of the light-cone sum rule does not exceed 35% and, simultaneously, the contribution from higher states estimated from duality is smaller than 40%. At $Q^2 > 1$ GeV^2, the predicted form factor $F^{\rho\pi}(Q^2)$ is reasonably stable under variations of the Borel parameter in the adopted ranges. At $Q^2 < 1$ GeV^2, the sum rule (26) becomes unstable signaling that one approaches too close to the physical region in the ρ - channel.

Figure 3 illustrates the sensitivity of $F^{\rho\pi}(Q^2)$ (at $M_{2pt}^2 = 0.7$ GeV^2) to the choice of nonasymptotic coefficients in $\varphi_\pi(u)$. We see that at $Q^2 \sim 10$ GeV^2, the difference between the form factors calculated with the asymptotic wave function and with the CZ or BF wave functions is quite substantial. The observed sensitivity to the nonasymptotic effects is due to the fact that at large Q^2, the integration over u in the light-cone sum rule is restricted to the end-point region, approximately, to the interval $1 - s_0/Q^2 < u < 1$. In this region, the integrals over nonasymptotic parts of the wave function (21) proportional to the Gegenbauer polynomials are of the same order as the integrated asymptotic part. The twist 4 contribution to (26) is between 35% and 10% at 1 GeV$^2 <$ $Q^2 < 10$ GeV^2. As already mentioned, this contribution is dominated by asymptotic wave functions and therefore

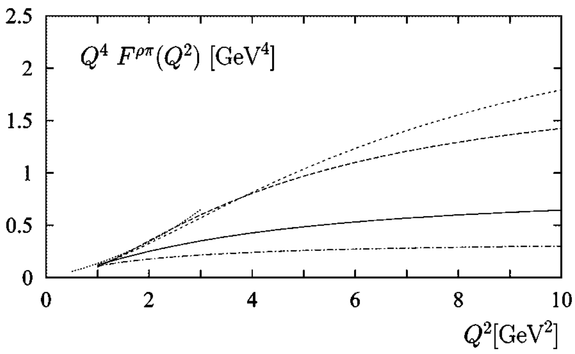

Fig. 3. $\gamma^*\rho \to \pi$ form factor calculated from the light-cone sum rule with the asymptotic pion wave function (solid), with the CZ wave function (long-dashed) and with the BF wave function (short-dashed), in comparison with the predictions of the three-point QCD sum rule (dotted) [24], and light-cone sum rule for the $\gamma^*\rho_\perp \to \pi$ form factor [8] (dash-dotted)

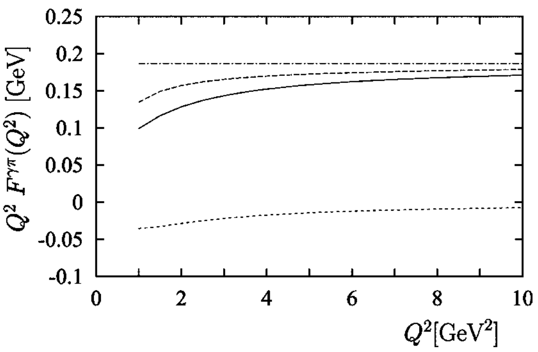

Fig. 4. Form factor of the $\gamma^*\gamma \to \pi^0$ transition calculated from the relation (27) with the asymptotic wave function of the pion (solid), twist 2 (dashed), twist 4 (dotted) contributions and the $Q^2 \to \infty$ limit (dash-dotted)

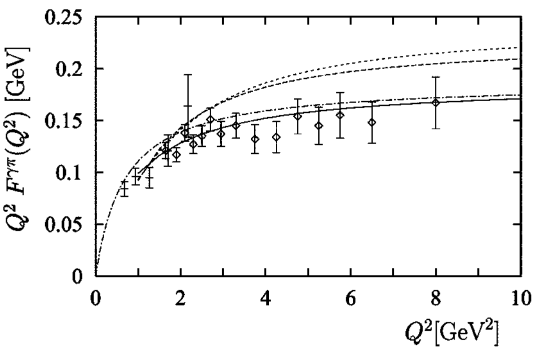

Fig. 5. Form factor of the $\gamma^*\gamma \to \pi^0$ transition calculated with the asymptotic (solid), CZ (long dashed) and BF (short-dashed) wave function of the pion in comparison with the experimental data points [9,22] and with the interpolation formula (35) from [1] (dash-dotted)

has a small uncertainty. We conclude that measurements of the $\gamma^*\rho\pi$ and $\gamma^*\omega\pi$ form factors at momentum transfer of order of a few GeV^2 can indeed be used to discriminate between various approximations for the twist 2 wave function $\varphi_\pi(u)$.

The form factor $F^{\gamma\pi}(Q^2)$ is calculated from the relation (27) with the same numerical input. In Fig. 4, it is plotted taking the asymptotic $\varphi_\pi(u)$ and $M^2_{2pt} = 0.7$ GeV^2. The twist 2 and 4 contributions are shown separately. We see a nontrivial Q^2-dependence of this form factor. At $Q^2 < 10$ GeV^2, it noticeably deviates from the asymptotic limit $Q^2F^{\gamma\pi}(Q^2) \to \sqrt{2}f_\pi$. Figure 5 shows the predictions on $F^{\gamma\pi}(Q^2)$ obtained with other choices of the twist 2 wave function. Starting from $Q^2 \simeq 3-4$ GeV^2, the role of the nonasymptotic part is quite essential.

The main uncertainty of the obtained predictions is due to the neglect of the perturbative α_s-correction and will be removed, once this correction is taken into account. The role of four-quark contributions such as (25), which are suppressed by extra powers of photon virtualities and α_s, cannot be important at $Q^2 > 1$ GeV^2. In order to estimate the accuracy of the leading-order approximation in α_s adopted here, the Borel parameter M^2_{2pt} was varied within $0.5 - 0.8$ GeV^2 and the threshold parameter s_0 within $1.3 - 1.8$ GeV^2. The resulting variations of $F^{\rho\pi}(Q^2)$ around the predictions shown in Fig. 3 are $\pm 5\%$ and $\pm 10\%$, respectively, almost independent of Q^2. The corresponding variations of $F^{\gamma\pi}(Q^2)$ are $\pm 3\%$ and $\pm 2\%$ at $Q^2 \sim 1$ GeV^2, and become negligibly small at larger Q^2. An additional uncertainty is connected with the choice of the normalization scale μ which is somewhat arbitrary in the absence of α_s-correcitons. Taking a Q^2-independent scale $\mu = 1$ GeV, which is of order of the Borel parameter, does not change the results at $Q^2 \sim 1$ GeV^2, but yields a 25% (10%) increase of $F^{\rho\pi}$ ($F^{\gamma\pi}$) at $Q^2 \sim 10$ GeV^2 in the case of the CZ and BF wave functions. The inclusion of the perturbative α_s-correction will certainly weaken this scale-dependence.

To have a more complete account of uncertainties of the method, one also has to assess the accuracy of the dispersion relation (3). In this relation, the isospin symmetry is assumed neglecting $\rho - \omega$ mixing and adopting (4) and (5). This is consistent with the isospin-symmetry limit of the light-cone OPE of the amplitude $F^{\gamma^*\pi}_{QCD}(Q^2, q^2)$. In addition, we adopt the zero-width approximations for ρ and ω. To clarify the sensitivity of form factors $F^{\rho\pi}$ and $F^{\gamma\pi}$ to these approximations, the resonance term in (3) has been modified to a finite-width Breit-Wigner form:

$$\frac{\sqrt{2}f_\rho F^{\rho\pi}(Q^2)}{m_\rho^2 + q^2} \tag{30}$$

$$\to \frac{1}{\sqrt{2}\pi}\sum_{V=\rho,\omega}\int\limits_{4m_\pi^2}^{s_0} ds \frac{m_V\Gamma_V f_V F^{V\pi}(Q^2)}{[(m_V^2 - s)^2 + m_V^2\Gamma_V^2](s + q^2)}\,.$$

substituting the experimental values [18] of $\Gamma_\rho = 151\,\mathrm{MeV}$, $\Gamma_\omega = 8\,\mathrm{MeV}$, $m_\omega = 782\,\mathrm{MeV}$, and $f_\omega \simeq 1/3(0.9)f_\rho$, and retaining $F^{\omega\pi}(Q^2) \simeq 3F^{\rho\pi}(Q^2)$. Numerically, the substitution (30) yields a 12% (6%) increase of $F^{\rho\pi}(F^{\gamma\pi})$. We use the magnitude of this change as a rough estimate of the combined uncertainty due to the resonant part in the dispersion relation (3).

Finally, for convenience, the obtained results on $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ form factors in the region $1 < Q^2 <$

$10\,\mathrm{GeV}^2$ have been fitted to the parametrizations

$$F^{\rho\pi}(Q^2)Q^4 = \frac{A^{\rho\pi}}{1 + \frac{B^{\rho\pi}}{Q^2} + \frac{C^{\rho\pi}}{Q^4}} \,, \tag{31}$$

and

$$F^{\gamma\pi}(Q^2)Q^2 = \frac{A^{\gamma\pi}}{1 + \frac{B^{\gamma\pi}}{Q^2}} \tag{32}$$

with

$$\begin{aligned} A^{\rho\pi} &= 0.92 \pm 0.2 \; (1.94 \pm 0.55) \; \mathrm{GeV}^4 \,, \\ B^{\rho\pi} &= 3.96 \; (2.27) \; \mathrm{GeV}^2 \,, \\ C^{\rho\pi} &= 2.48 \; (13.5) \; \mathrm{GeV}^4 \end{aligned} \tag{33}$$

and

$$\begin{aligned} A^{\gamma\pi} &= 0.186 \pm 0.02 \; (0.242 \pm 0.04) \, \mathrm{GeV} \,, \\ B^{\rho\pi} &= 0.875 \; (1.385) \, \mathrm{GeV}^2 \,. \end{aligned} \tag{34}$$

The numerical values of the above parameters correspond to the asymptotic (CZ) choice of the pion light-cone wave function. The quoted normalization errors (conservatively) take into account the estimated theoretical uncertainties of the leading-order approximation in α_s considered in this analysis.

5 Conclusion

In this paper, the $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ form factors have been calculated using the light-cone OPE, the dispersion relation and the quark-hadron duality in the ρ-meson channel. The main results are in (26) - (29), expressing $F^{\rho\pi}(Q^2)$ and $F^{\gamma\pi}(Q^2)$, respectively, in terms of light-cone wave functions of the pion. At Q^2 of order of a few GeV^2, the numerical predictions on both form factors are sensitive to nonasymptotic effects in the twist 2 wave function $\varphi_\pi(u)$.

In Fig. 5, the obtained results for the form factor $F^{\gamma\pi}(Q^2)$ are compared with the new CLEO data [9] and with the earlier CELLO data [22]. This comparison supports the asymptotic form of the wave function $\varphi_\pi(u)$. More definite quantitative conclusions can be made after including perturbative corrections in our analysis. Note that the $\gamma^*\rho \to \pi$ form factor can also be measured, e.g. by extracting the one-pion exchange in the electroproduction of ρ, ω mesons [23].

In Fig. 3, our prediction on $F^{\rho\pi}(Q^2)$ is compared with the results of other calculations. In [8], a light-cone sum rule for the $\gamma^*\rho_\perp \to \pi$ transition form factor has been obtained from a correlation function of two currents, j_μ and $\bar{d}\sigma_{\mu\nu}u$ ($\rho_\perp$ is a ρ-meson with the helicity $\lambda = \pm 1$). The leading contribution to this sum rule is generated by the twist 3 wave function of the pion. The higher-twist terms are not known, hence, the achieved accuracy is not high. Therefore, only a crude agreement with our prediction obtained with the asymptotic $\varphi_\pi(u)$ can be expected. Figure 3 also shows the $\gamma^*\rho \to \pi$ form factor obtained [24] from the three-point sum rule[3] in the region $Q^2 = 0.5 - 3\ \mathrm{GeV}^2$. The three-point sum rule prediction is in a good agreement with our result obtained with the CZ and BF wave functions. The latter result also agrees with the form factor $F^{\rho\pi}(Q^2)$ calculated in the relativistic quark model [25] in the same region. Furthermore, the form factor $F^{\rho\pi}(Q^2)$ obtained in the light-front constituent quark model [26] at $Q^2 = 1{-}8\ \mathrm{GeV}^2$, is quite close to our prediction obtained with the asymptotic $\varphi_\pi(u)$.

Turning to the $\gamma^*\gamma \to \pi^0$ transition, we see from Fig. 5 that at $Q^2 > 1\ \mathrm{GeV}^2$ the relation (27) is in a good numerical agreement with the simple interpolation formula

$$F^{\gamma\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{4\pi^2 f_\pi^2 + Q^2} \,, \tag{35}$$

suggested in [1], if the asymptotic $\varphi_\pi(u)$ is adopted. In [10], the form factor $F^{\gamma\pi}(Q^2)$ was calculated using 3-point correlation function, short-distance OPE and QCD sum rule in the pion channel. The long-distance interaction of the small virtuality photon was taken into account introducing bilocal correlators, employing duality and light-cone wave functions. After that, $F^{\gamma\pi}(Q^2)$ has been obtained in terms of a combined nonperturbative input including quark/gluon condensates and light-cone wave functions of ρ -meson and photon. Numerically, the result of [10] is close to the interpolation formula (35) and therefore also to our prediction for the $\gamma^*\gamma \to \pi^0$ form factor obtained with the asymptotic wave function of the pion.

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